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# SOFTWARE TOOL FOR CALCULATING THE VOLUME OF THE TETRAHEDRON ON THE LENGTHS OF ITS EDGES

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*This paper describes the work of the software "calculator" that can be used to calculate the volume of the tetrahedron on the lengths of its edges.*

***Keywords:*** *pyramid, volume, tetrahedron, Jungius, calculator.*

It is well known condition for the construction of a triangle from three segments: if the length of each of the three segments is less than the sum of the lengths of the other two segments, then these segments is possible to construct a triangle, and conversely, the length of each side of the triangle less than the sum of the lengths of the other two sides of a triangle (the triangle inequality).

Another analogue of this condition can serve as a condition of existence of nonzero area of the triangle, which is calculated from the lengths of its three sides. This area can be calculated from the known Herons formula:

$s = \frac{1}{4}\sqrt{(a+b+c)(b+c-a)(a+c-b)(a+b-c)}$, where $a$, $b$, $c$ – sides a triangle.

This formula shows that for the existence of a nonzero area of a triangle if and only if each of its sides was less than the sum of the other two sides.

Tetrahedron (triangular pyramid) is the simplest polyhedron. Euclid called pyramid geometric solid, lying between the planes and is posed from one plane to one point.

At the time of Plato's polyhedron seen as empty geometric solid, consisting of some edges. Aristotle distinguished between empty and filled with polyhedral and treated them as different body. Euclid considered as filled polyhedron of the body, but he did not specify what they are filled with, since the antique mathematicians did not use the formal concept of space [1, p. 164].

Authors do not know the conditions of the construction of a tetrahedron on the lengths of its edges. As one of these conditions can serve as a non-zero volume of a tetrahedron whose edges are given six segments. German mathematician Joachim Jungius (1587-1657) received formula for the volume of the tetrahedron on the lengths of its edges [2, c. 100]. But the construction of a tetrahedron of the given segments, as shown by concrete examples, can lead to different values of the volume of a tetrahedron, and in some cases, the tetrahedron cannot exist. Finding the volume of a tetrahedron with different permutations of the edges leads to a large Number of calculations of a formula Jungius (720 different permutations of the given six segments). In this paper describes the authors developed a calculator that calculates the volume of the tetrahedron cycles, with different permutations of its edges. By result calculation of the calculator can be seen on the existence of a tetrahedron and get its volume, if it exists.

To write the formula Jungius need to introduce the following notations tetrahedral elements.

Given a tetrahedron *SABC* (figure 1). The edges of the tetrahedron we denote:

$$AB = a_1,\ AS = a_2,\ AC = a_3,\ BS = a_4,\ BC = a_5,\ CS = a_6.$$

We denote volume of the tetrahedron – $v$.

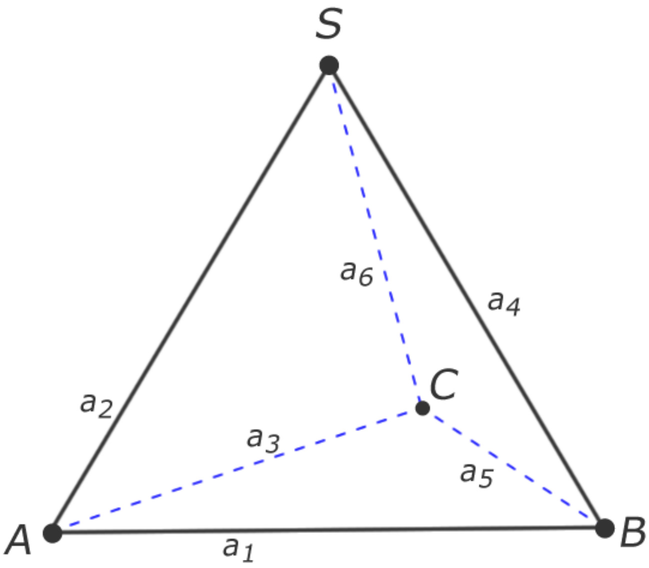


*Figure 1. The figure shows a tetrahedron and its elements.*

If this notation is given by the formula Jungius is:

$$v^2 = \frac{1}{144}(a_1^2 a_6^2(a_2^2 + a_3^2 + a_4^2 + a_5^2 - a_1^2 - a_6^2) +$$

$$+a_2^2 a_5^2(a_1^2 + a_3^2 + a_4^2 + a_6^2 - a_2^2 - a_5^2) + a_3^2 a_4^2(a_1^2 + a_2^2 + a_5^2 + a_6^2 - a_3^2 - a_4^2) -$$

$$-a_2^2 a_3^2 a_6^2 - a_1^2 a_3^2 a_5^2 - a_1^2 a_2^2 a_4^2 - a_4^2 a_5^2 a_6^2).$$

Examples of specific numerical values show that for the same segments, with their specific permutations of them, can build a tetrahedron, while other permutations cannot.

For some segments of three of the six specified may not be satisfied the triangle inequality (therefore, one can construct a triangle), but the volume of a tetrahedron can exist in this case.

Such as:

$a_1 = a_3 = a_5 = 1$, $a_2 = a_4 = a_6 = 3$ using the formula Jungius, we find:

$v = \frac{\sqrt{26}}{12}$. In this case, the inequality $a_2 = 3 > a_1 + a_3 = 2$ is satisfied.

On the other hand, if the triangle inequality holds for any three of the six segments defined, then the tetrahedron with such edges may not exist.

Such as:

$a_1 = a_3 = a_5 = 1$, $a_2 = a_4 = \frac{\sqrt{3}}{3}$ and $a_6 = \frac{1}{2}$ then the value of the right side of formula Jungius will be negative:

$v^2 = -\frac{13}{144^2}$.

In this case, the triangle inequality holds for all three segments of the six specified, since it holds for the maximum length segment $a_3$ and for the two smallest length $a_2$ and $a_6$ segments.

Indeed, we find:

$a_2 + a_6 = \frac{\sqrt{3}}{3} + \frac{1}{2} \approx 1{,}07$. But $a_3 = 1$, then $a_2 + a_6 > a_3$.

In addition, the same segments can be constructed tetrahedron with different volumes.

Such as:

$a_1 = a_3 = a_5 = 2$, $a_2 = a_4 = a_6 = 3$.

Using the formula Jungius, we find:

$v = \frac{\sqrt{23}}{3}$.

If we take the opposite:

$a_1 = a_3 = a_5 = 3$, $a_2 = a_4 = a_6 = 2$, then $v = \frac{3\sqrt{3}}{4}$.

From these examples, the conclusion is that the volume of the tetrahedron depends on its orientation.

To test the feasibility of constructing a tetrahedron of the given six segments can be used by formula Jungius. It needs to be numbered segments in accordance with sections made in this paper the notation and calculate the right side of formula Jungius. If right side of the formula is not positive, then the tetrahedron of the numbered sections so you cannot build. If right side of the formula is positive, then the tetrahedron can be constructed.

The complexity of this method consists in a large number of permutations of the six segments -720. For a large number of calculations can be used authors special calculator. This calculator calculates the right side of formula Jungius for all possible permutations of the six segments. With the work of the calculator, which is described in this paper, can be found at the following address: http://ksuonline.ksu.ks.ua/mod/resource/view.php?id=2645

Figure 2 shows the working field calculator. On the calculator you can get all the possible values of the right side of the formula Jungius, all positive, all zero, all negative values of the formula.

To calculate the value of the formula Jungius need to enter values edges $a_1, a_2, a_3, a_4, a_5, a_6$ of the tetrahedron in the right side of the working field calculator. Then choose the desired set of values of formula (all values, positive values, zero, negative) and button «calculation» to activate (figure 2). When it is done in the working field calculator will appear «The calculation is complete». In the working field calculator will show the selected values of the formula Jungius and number of all calculated values. If the value of the formula will be zero or negative, the calculator will indicate that tetrahedron cannot exist. If the value of the formula will be positive, the calculator will indicate the value volume of the tetrahedron. In the working field of the calculator the value of a volume tetrahedron is denoted by «v», and the value of a square volume is denoted by «v2».

If data is entered incorrectly, the calculator displays the appropriate message. If the data field «an» is empty, a message appears «Input the value an». If the data field «an» is not filled with the numerical value «s», or a fractional number is written with a comma, a message appears «The value of «an» is incorrect (s). Use point instead of commas for floating».


*Figure 2. All operating buttons are on-screen calculator.*

Figure 3 shows the positive, zero and negative values square volume of a tetrahedron.

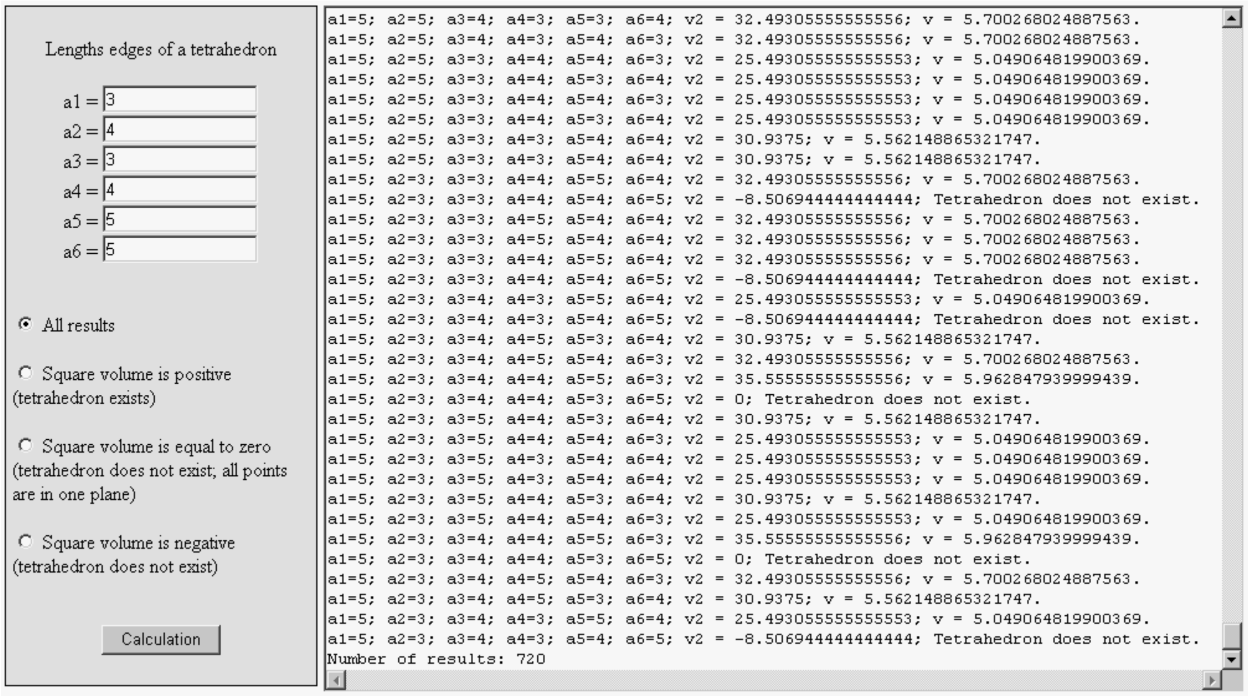


*Figure 3. The screen shows all the results of the calculations.*

Figure 4 shows only the positive values of the formula. In these cases, there is a tetrahedron.

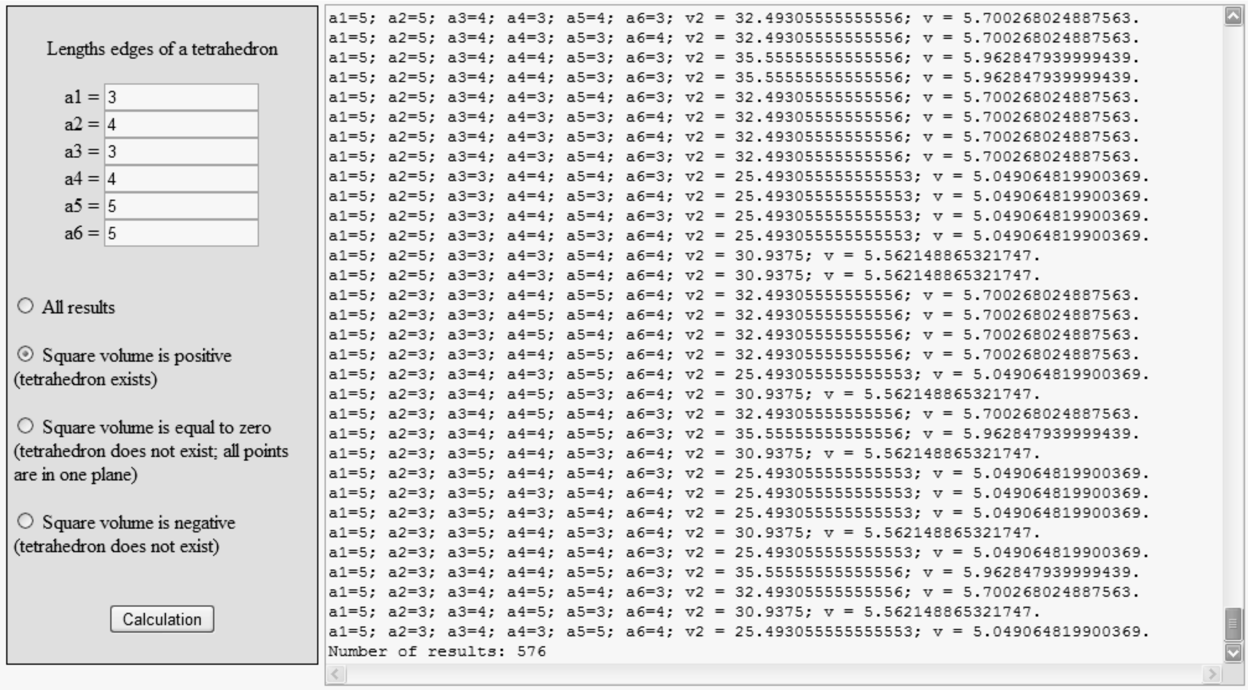


*Figure 4. Square volume is positive.*

Figure 5 shows only zero values square volume of a tetrahedron. In this case, its volume is also equal to zero, therefore tetrahedron does not exist (it is impossible to construct) and all six points lie in one plane.

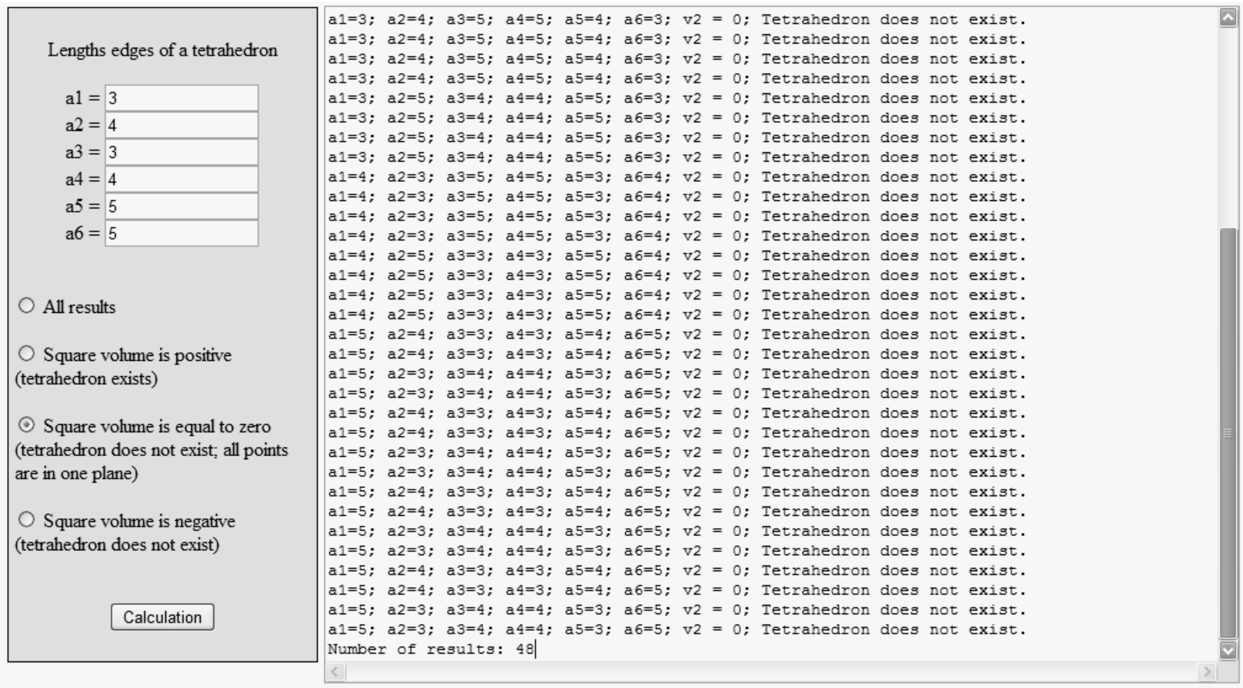


*Figure 5. Square volume is zero.*

Figure 6 shows only negative values square volume of a tetrahedron. In these cases, tetrahedron do not exist (it is impossible to construct).

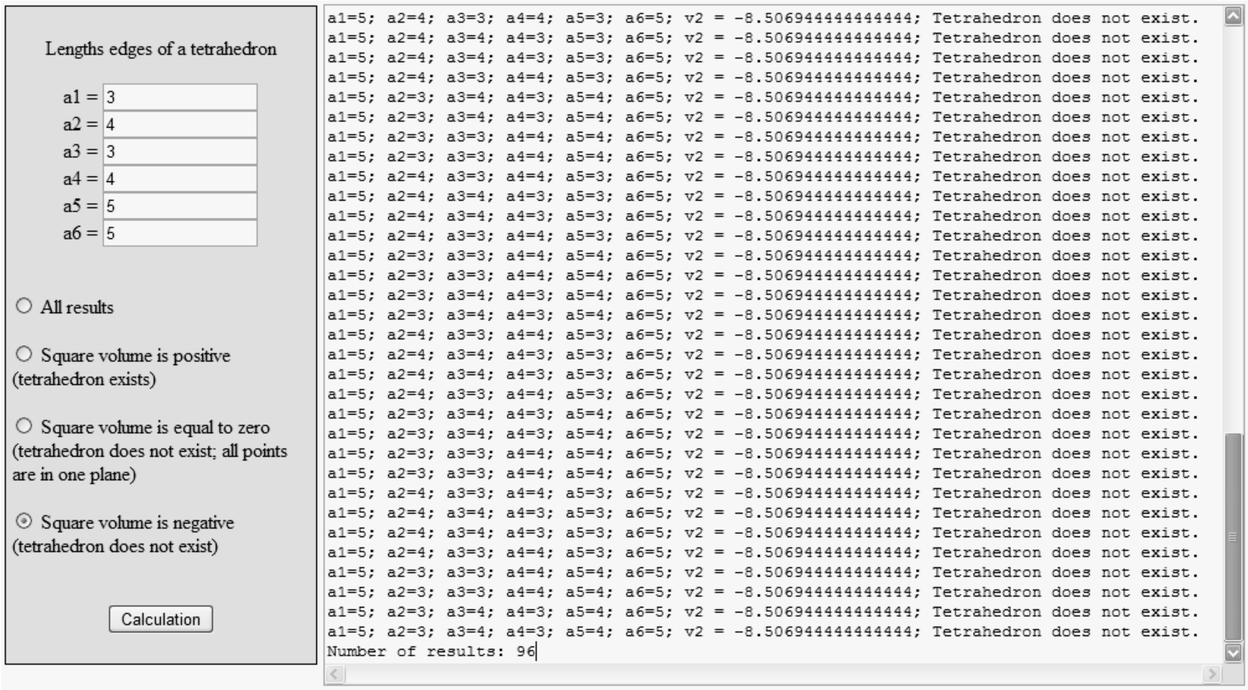


*Figure 6. Square volume is negative.*

Work calculator with specific numeric values indicates that the orientation of a tetrahedron in space influence its existence and its characteristics.

It is known that any of two polygons of equal squares can be dissected into a finite number of polygons, from which we can draw another polygon. For two polyhedron equal volumes this property is not always satisfied. It is proved M. Dehn [3, p. 6]. This he proved third Hilbert's problem [4, p. 28]. So in the future this work can be used to create these calculators for other figure in space, which are composed of a finite number of tetrahedrons.

Calculator is presented in Ukrainian, Russian and English versions. The authors thank Boukoulou Didier Criss and Alferov E. for help in translating into English the Calculator, and for adapting the Calculator on the website of the Kherson State University.

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